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RESEARCH ARTICLE

ENHANCING THE PROBLEM-SOLVING AND CRITICAL THINKING SKILLS OF STUDENTS USING THE MATHEMATICAL INVESTIGATION APPROACH

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Abstract

The interest in understanding what is involved in doing mathematical investigations brought together several researchers and classroom teachers. In this paper, the proponents set up a project to experiment with tasks involving students exploring and investigating mathematical ideas, concepts, and processes. The work includes producing, experimenting, and evaluating such tasks and studying the competencies needed for using them successfully in the mathematics classroom. This study aimed to describe the attitude and performance of the students when exposed to investigative tasks. It is also driven to compare the performance of the control and experimental groups regarding the implementation of the intervention and to see whether their attitude towards math investigation matters in the students' problem-solving and critical thinking skills. The findings showed that on the attitudes, the experimental group had a very favorable attitude toward the investigation process despite its complexity, particularly in giving proof. The data obtained claimed that the control group did meet the expectations for both tests. At the same time, the experimental post-test presented a very satisfactory performance indicating that their exposure to investigations taught them to explore and provide absolute mathematical reasoning. A significant difference was found in the performance of the two groups making the experimental group better than the control group, as reaffirmed in the large effect size. Further, students' attitudes on implementing math investigation are described as a predictor of successful problem solving and improving the students' critical thinking skills.

INTRODUCTION

Teaching and learning mathematics has always been considered the most challenging academic activity for teachers and learners (Bacsal et al., 2022; Pentang et al., 2020). This is observed from the gaps that divert the bond between effective teaching strategy and the ability of the learners to digest what has been taught. The teaching principle asserts that effective mathematics teaching begins with a clear understanding of what students already know and need to learn, followed by a challenge and support system to help them master it (Mariano-Dolesh et al., 2022; Santos et al., 2022). It suggests that one must have to propagate the knowledge of the learners and the knowledge of the teaching pedagogies. Developing learners' mathematical abilities allows teachers to assess their instructional practices' positive impact. Similarly, teachers' knowledge of appropriate pedagogies will provide them with various instructional techniques that can be used and shared in any given situation (Pentang et al., 2020; Pentang, 2021), especially in this new normal.

Implementing the Mathematics Program in SDO Isabela has been aligned with the twin goals of helping learners' critical thinking and problem-solving skills. In its unending pursuit for excellence in Mathematics education, the teachers have always thought of various programs, projects, and activities anchored to the educational reform of the K to 12 Program upon the enactment of Republic Act 10533, known as the Enhanced Basic Education Act 2013. However, the unprecedented pandemic has thwarted implementing the planned activities to address teaching and learning needs, as shown by Agayon et al. (2022), Bacomo et al. (2022), Caasi and Pentang (2022), Hamora et al. (2022), and Zakaria et al. (2022). Thus, revisions and adjustments to the mode of delivery and implementation have been made to adopt programs, projects, and activities in the context of the New Normal and aligned with the Basic Education Learning Continuity Plan of SDO Isabela. In support of DepEd's Project K.I.T.E, mathematics teachers in SDO Isabela have to undergo continuous professional advancement from the identified training or programs aimed at improving mathematics quality and teacher practice, including training on Mathematical Investigation. Thus, this study made use of the Mathematical Investigation approach in teaching and learning math, where students are tasked to broaden and expand their knowledge by posing endless questions/conjectures and able to test them using previous ideas such as theorems, definitions, and other facts they already know.

Research Questions

This mainly aimed to answer the following questions:

1. What is the student's attitude toward integrating the Mathematical Investigation (MI) approach as an instructional strategy?
2. What is the student's performance in the experimental and control groups before and after integrating the MI approach in math class?
3. Is there a significant difference in the student's performance before and after the integration of MI within each group?
4. Is there a significant difference in the student's performance before and after the integration of MI between groups?
5. Is there a relationship between students' attitude toward MI and their performance after exposure?
6. What is the effect size or magnitude difference of using the MI approach on students' performance?

METHODOLOGY

This study employed the actual experimental research design, particularly the pre-test and post-test, with two influential groups called the experimental and control groups. This research was conducted at Regional Science High School for Region 02 for 2021-2022. The study's respondents were Grade 11 students, composed of 80 students who served as official samples after having accomplished some required criteria of the paper. They were selected using the purposive sampling technique with the identified standards or criteria: a) Learning Modality, b) Proportionality of samples, and c) Complete Participation Rate or Maturation. The categorization of the said respondents was based on the learning modality of the target participants; that is, students who participated in the synchronous online class were classified as the experimental group, while those in the pure modular class were assigned the control group.

The researchers integrated the MI process into their Weekly Home Learning Plan (WHLP) and duly signed and concurred by the school head. The data gathered were mainly taken from the outputs of the respondents in five (5) investigative tasks. Each investigative task was appropriated within one week to complete the task. This gives the students and teachers two (2) months to gather information. During this period, the teachers provide feedback after the submission to discuss the investigative task and its process. This is done virtually, where the experimental students present their outputs orally while teachers give additional inputs or insights to improve the MI report further.

A transmutation table of scores and their equivalent grades was adopted from Gladys C. Nivera and Wilfred C. Bagsao, the EPS, Math (SDO-Benguet). This rubric comprises 15 general criteria with a four-point scale giving a total score of 60 per investigative task. Before selecting the appropriate statistical tools, the researchers first sought the test for the data's normality to identify whether the respondents' scores are normally distributed or approximate bell curve or symmetric.

Shapiro-Wilk test, Skewness, and Kurtosis were used for the normality test, which showed that data points are approximately normal. To test the inferential question on significant difference, the researchers use Paired Sample t-test for within groups and an unrelated/independent t-test for between groups at 0.05 alpha along with standard deviation and mean average. Moreover, Partial Eta Square η_p^2 and Cohen's D guideline for Paired Sample t-test is applied to quantitatively describe the effect size or magnitude difference of students' learning on Mathematical Investigation under the experimental group. Pearson-r was employed for the correlation of attitudes and performance.

RESULTS AND DISCUSSION

Students' Attitude toward the Integration of the Mathematical Investigation (MI) Approach as an Instructional Strategy

Table 1 presents the students' attitude in the experimental group in the integration of mathematical investigation in teaching some concepts in Grade 11 math lessons. It is observed that students have a **very favorable** attitude towards math investigation, particularly in enhancing their confidence by expressing their math ideas freely, helping them employ the methods of explorations, formulating and testing conjectures, giving proofs, and finally, teaching them to communicate/report their conceptual understanding with the mean ratings of 3. 42, 3.55 and 3.30, respectively. Furthermore, the students have a favorable attitude along with the remaining indicators.

The grand mean of 3.16 suggests that the experimental group has a **favorable** attitude when the teachers apply the investigation process in their instructions. This implies that despite mathematical reasoning and proof rigor, students still appreciate that math investigation widens their ingenuity and mathematical horizon. Similar to Flewelling and Higginson (2005), Mariano-Dolesh et al. (2022), and Pentang (2019), inquiry, investigation, and problem-solving allow learners to employ their imagination and develop the habit of doing so.

Table 1. Students' attitude toward the integration of the mathematical investigation

INDICATORS	MEAN	SD	Description
1. MI promotes the use of critical thinking necessary in mastering mathematics competencies.	2.83	0.84	Favorable
2. Presentation of outputs in hands-on activities enhances confidence in expressing ideas in Mathematics.	3.42	0.71	Very Favorable
3. MI Approach is a strategy that leads to meaningful problem-solving rather than rote memorization.	3.00	0.78	Favorable
4. This method highlights the process of making conjectures, explorations, and justifying observations.	3.55	0.60	Very Favorable
5. This technique focuses on a student-centered approach and active learning, with the teacher acting as a facilitator of learning to gain a more meaningful content understanding of the topics.	3.10	0.81	Favorable
6. This teaching method supports students' independence and self-worth in learning.	3.23	0.80	Favorable
7. Students build a strong foundation in technical, conceptual, and communications skills through Math Investigation.	3.30	0.76	Very Favorable
8. This approach teaches students to demonstrate creativity and authentic learning experiences.	3.15	0.92	Favorable
9. This approach develops students' innovative skills and ability to apply conceptual understanding to real-life practices.	2.98	0.86	Favorable
10. Lastly, this model permits students to improve critical thinking and proving skills,	3.03	0.83	Favorable
Grand Mean	**3.16**	**0.29**	**Favorable**

Students' Performance of the Experimental and Control Groups Before and After the Mathematical Investigation

Table 2 shows the results of each group's pre and post-test scores from the two activities as pre-test and five investigative tasks for the post-test. The average score expressed as a mean is transmuted by applying the transmutation table provided by Gladys C. Nivera in "Mathematical Investigation and Its Assessment: Implications for Mathematical Teaching and Learning". It reveals in the test mean scores that the pre-test and post-test of the control group are 20.6 and 21.63 and transmuted as 69.0 and 70.0, indicating that the students **did not meet the expectations** of enhancing their problem-solving and critical thinking skills. Evidently, the students did not apply the correct rules or follow the established rules, as per Mariano-Dolesh et al. (2022), Pentang (2019), and Santos et al. (2022).

On the other hand, the performance of the students in the experimental group during the pre-test **did not meet expectations,** with a transmuted score of 74.0. At the same time, the post-test result presents a score of 88.0, indicating **very satisfactory** investigation skills of students, which leads to improved problem-solving and critical thinking skills. This supports further that integrating the math investigation in math discussion promotes a better conceptual understanding of solving open-ended problems. NCTM (2000) asserts that in learning problem-solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the math classroom.

Table 2. Performance of the experimental and control groups before and after the mathematical investigation

Test	Mean	Transmuted Score	Description
Control Group			
Pre-Test	20.60	69.0	Did Not Meet Expectations
Post-Test	21.63	70.0	Did Not Meet Expectations
Experimental Group			
Pre-Test	28.70	74.0	Did Not Meet Expectations
Post-Test	46.34	88.0	Very Satisfactory

Comparison between the Pre-test and Post-test Scores of the Control Group

Table 3 presents the comparative analysis of the pre and post-test scores of the control group, which divulges that both test results obtained a t-value of 2.39, the probability *value* of 0.2 and the effect size $\eta_p^2 = .23$. This explains that since the $p > .05$, then the pre-test and post-test scores are statistically insignificant. This means that students' performances do not differ before and after, which can be supported by the description of their problem-solving skills as not meeting expectations.

Table 3. Comparison between the pre-test and post-test scores of the control group

Test	Mean	SD	t(39)	p	η_p^2
Pre-Test	20.6	2.37	2.39	.20	.23
Post-Test	21.63	1.32			

Comparison between the Pre-test and Post-test Scores of the Experimental Group

Data in table 4 provide the comparative analysis of the pre-test and post-test from the experimental group. The table reveals that the $p = .000 < .05$ which means that the before and after performance of the students are statistically significant. The findings show that the application of the treatment/intervention, which is the math investigation, has a significant effect on students' problem-solving skills and the activation of their critical thinking, agreeing with Pentang (2021), where interventions can help improve the students' mathematical performance.

The math investigation process requires systematic exploration of open situations with mathematical characteristics, which requires using higher-order thinking skills rather than memorizing basic math facts. The outcome is consistent with the premise that "investigations are central to the internationally advocated reforms to improve mathematical learning and develop children's mathematical ability" (Baroody & Coslick, 2017).

Table 4. Comparison between the pre-test and post-test scores of the experimental group

Test	Mean	SD	t(39)	p	η_p^2
Pre-Test	28.78	2.54			
Post-Test	46.34	0.93	41.57	.000	.99

Comparative Analysis of the Pre-test Scores of the Control and Experimental Group

Table 5 shows the analysis of the pre-test results of the two groups, and data present that $p = .332 > .05$ indicating that the two scores are insignificant or have differences. This suggests that the students' performances are equal, which satisfies the essential requirement for homogenizing the experimental and control groups before the intervention. This, therefore, denotes that the groupings of the students are uniformly distributed or classified.

Table 5. Comparative analysis of the pre-test scores of the control and experimental groups

Test	Mean	SD	t(78)	p	η_p^2
Experimental	28.78	2.54	1.44	.332	.16
Control	20.60	2.37			

Comparison between the Post-test Scores of the Control and Experimental Groups

Table 6 shows the comparative analysis of the post-test scores of the experimental and control groups, which shows a probability value of .000 and is less than the alpha of 0.05, this means that there is a significant difference in the student's performance relative to the implementation of the intervention called the math investigation. This implies that those in the experimental group have a better understanding or skills in solving math problems it is because the concept of math investigation is leading the learners to always think outside the box, see math differently, and is not mimicking merely the role of teachers as the sole source of information. Wood (2014) asserts that investigations assist in establishing a dynamic learning environment in an inquiry math program.

Moreover, Oliviera et al. (2017) emphasized that math investigation (a) encourages the kind of student engagement necessary for meaningful learning; (b) offers a variety of entry points for students of various skill levels; (c) encourages a holistic mode of thought, relating many topics, a necessary prerequisite for meaningful mathematical reasoning; and (d) is essential to providing a comprehensive view of mathematics because they are a necessary component of mathematical activity.

Table 6. Comparison between the post-test scores of the control and experimental groups

Test	Mean	SD	t(78)	p	η_p^2
Experimental	46.34	0.93	96.95	.000	.995
Control	21.63	1.32			

Effect Size of Using the Mathematical Investigation Approach on Students' Performance

It is shown in the above table that using the approach of math investigation; there is a significant effect size of **large** as indicated in the partial eta squared of .99. This finding can be associated with the fact that investigations offer a helpful framework for teaching students that their claims must be supported by evidence, express their reasoning and proofs and communicate their findings based on mathematical facts such as theorems, lemmas, and propositions. Hence, continuous use of math investigation could help learners find the authentic meaning of mathematics in real-life situations and the need to learn to think for themselves.

Table 7. Effect Size of using the MI approach on students' performance

Test	η_p^2	Description
Control Group		
Pre-Test	.23	Small
Post-Test		
Experimental Group		
Pre-Test	.99	Large
Post-Test		

Relationship between the Students' Attitude in Math Investigation and their Performance
Table 8 discusses the correlation between students' attitude toward math investigation and their performance in doing investigative tasks; it presents that the $p = .913 > .05$, explaining that attitude is a predictor that contributes to **very satisfactory** performance of the students in five investigative tasks after learning the processes and stages of exploring math problem. The analysis $(\mathrm{r} = 0.718)$ suggests that there is a **high positive correlation** between attitudes and performance which means that the student's preference to learn math investigation and ability to apply critical thinking skills in problem-solving are related to their conceptions, knowledge, competencies, and values. This result opposes Ibañez and Pentang (2021), where conceptual understanding is indirectly associated with attitude.

Table 8. Relationship between the students' attitude in MI and their performance

Variables	Pearson's r	p-value
Exp. Post Test Attitude	.718	.913

CONCLUSION AND RECOMMENDATION

Valid and engaging mathematical tasks must be the foundation of effective mathematics instruction. Suppose the suggested mathematical assignments are unsuitable for a rich mathematical study and must be sufficiently challenging. In that case, more than the teacher's ability to develop a stimulating learning environment and provide numerous opportunities for student debate and reflection will be required. Students' mathematics experiences may require investigations, and the shelves of instructional resources may be packed with ideas.

The teacher, however, will always play a vital role in this process, and some of their work cannot be formalized. Resting in his or her mathematical and educational sense, selecting what matters at each moment, listening intently, being adaptable, and attempting to determine the best course of action for the next step are all necessary. The mix of a deliberate decisions, expert knowledge, and artistic ability distinguishes teaching as a profession and makes teaching mathematics a hugely challenging endeavor.

Based on the findings and discussions, the following suggestions are deemed necessary.

1. Introduce students to an intensive process of showing proofs in various methods.
2. Teach students to explore the use of technology in generating recursive values.
3. Expose students to communicate the final report of their investigative outputs.
4. Similar studies are advised to be conducted to verify the results of the present study.

REFERENCES

Agayon, A. J. D., Agayon, A. K. R., & Pentang, J. T. (2022). Teachers in the new normal: Challenges and coping mechanisms in secondary schools. *International Journal of Humanities and Education Development, 4*(1), 67-75. https://doi.org/10.22161/jhed.4.1.8

Bacomo, A. P. C., Daculap, L. P., Ocampo, M. G. O., Paguia, C. D., Pentang, J. T., & Bautista, R. M. (2022). Modular learning efficiency: Learner's Attitude and performance towards self-learning modules. *IOER International Multidisciplinary Research Journal, 4*(2), 60-72. https://doi.org/10.5281/zenodo.6511718

Bacsal, E., Ibañez, E., & Pentang, J. (2022). Jigsaw strategy: Strengthening achievement and interest in mathematics among elementary pre-service teachers. *Palawan Scientist, 14*(1), 35-42. https://www.palawanscientist.org/tps/jigsaw-strategy-strengthening-achievement-and-interest-in-mathematics-among-elementary-pre-service-teachers/

Caasi, N. B., & Pentang, J. (2022). Parental factors related to students' self-concept and academic performance amid COVID-19 and distance learning. *Universal Journal of Educational Research, 1*(4), 202-209. https://doi.org/10.5281/zenodo.7270542

Diezmann, C. M. (2005). Challenging mathematically gifted primary students. *Australasian Journal of Gifted Education, 14*(1), 50-57. Retrieved 2 April 2009 from http://search.informit.com.au.ezproxy.cdu.edu.au/fullText;dn=154474;res=AEIPT

Diezmann, C. M., Watters, J. J., & English, L. D. (2011). Difficulties confronting young children undertaking investigations. Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education. Utrecht. Retrieved 3 April 2009 from http://eprints.qut.edu.au/1853/1/1853.pdf

Flewelling, G. & Higginson, W. (2005). Teaching with rich learning tasks: A handbook (2nd ed.). Adelaide: Australian Association of Mathematics Teachers.

Hamora, L., Rabaya, M., Pentang, J., Pizaña, A., & Gamozo, M. (2022). Students' evaluation of faculty-prepared instructional modules: Inferences for instructional materials review and revision. *Journal of Education, Management and Development Studies, 2*(2), 20-29. https://doi.org/10.52631/jemds.v2i2.109

Ibañez, E., & Pentang, J. T. (2021). Socio-constructivist learning and teacher education students' conceptual understanding and attitude toward fractions. *Indonesian Research Journal in Education, 5*(1), 23-44. https://doi.org/10.22437/irje.v5i1.12187

Leach, S. & Bowling, J. (2010). A classroom research project: ESL students and the language of mathematics. *Australian Primary Mathematics Classroom, 5*(1), 24–27

Lovitt, C. & Lowe, I. (2014). *Mathematics curriculum and teaching program. Chance and data investigations* (Vol. 1). Melbourne: Curriculum Corporation

Mariano-Dolesh, M. L., Collantes, L. M., Ibañez, E. D., & Pentang, J. T. (2022). Mindset and levels of conceptual understanding in the problem-solving of preservice mathematics teachers in an online learning environment. *International Journal of Learning, Teaching and Educational Research, 21*(6), 18-33. https://doi.org/10.26803/ijlter.21.6.2

Oliveira, H., Segurado, I., Ponte, J. P., & Cunha, M. H. (2017). Mathematical investigations in the classroom: A collaborative project. In V. Zack, J. Mousley, & C. Breen (Eds.), Developing practice: Teachers' inquiry and educational change (pp. 135-142). Geelong: Centre for Studies in Mathematics, Science and Environmental Education.

Pentang, J. (2019). *Determining elementary pre-service teachers' problem solving performance and skills through sequential explanatory approach* [Master's thesis, Central Luzon State University]. DOST Union Catalog. https://philpapers.org/rec/PENDEP

Pentang, J. T., Bautista, R. M., Pizaña, A. D., & Egger, S. P. Mathematical needs of Laura Vicuña learners. (2020). *Western Philippines University Graduate School, 5*(1), 78-81. http://rgdoi.net/10.13140/RG.2.2.12130.66241

Pentang, J. (2021). Impact assessment and clients' feedback towards MATHEMATICS project implementation. *International Journal of Educational Management and Development Studies, 2*(2), 90-103. https://doi.org/10.53378/346107

Santos, R. A., Collantes, L. M., Ibañez, E. D., Ibarra, F. P., Pentang, J. T. (2022). Innate mathematical characteristics and number sense competencies of junior high school students. *International Journal of Learning, Teaching and Educational Research, 21*(10), 325-340. https://doi.org/10.26803/ijlter.21.10.18

Silver, E. A. (2013). On mathematical problem posing. Proceedings of PME 15 (Vol I, pp. 66-85), Tsukuba, Japan.

Tymoczkco, T. (2014). New directions in the philosophy of mathematics. Boston: Birkhauser.

Wood, T. (2014). Patterns of interaction and the culture of the mathematics classroom. In S. Lerman (Ed.), Cultural perspectives on the mathematics classroom (pp. 149- 168). Dordrecht: Kluwer.

Zakaria, W., Turmudi, T., & Pentang, J. (2022). Information and communication technology in elementary schools: A comparison between hybrid and face-to-face learning systems. *Profesi Pendidikan Dasar,* 9(1), 46-54. http://dx.doi.org/10.23917/ppd.v9i1.17534